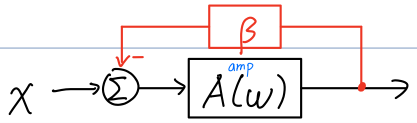
Feedback

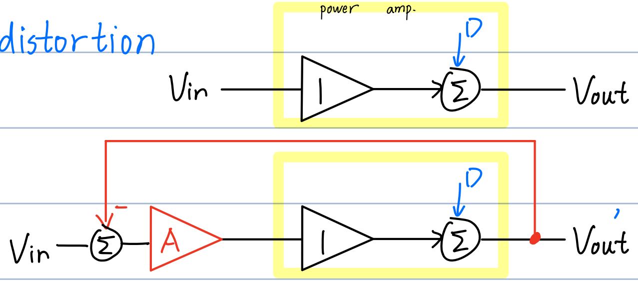


$0 < \beta \leq 1$ (E. v. divider)

$$y \overset{\cdot\cdot\cdot}{=} \frac{A}{1+A\beta} x$$

E. $G=2 \rightarrow \beta = \frac{1}{2}$, $y \approx 2x$, indep. of $A >> 1$

E. distortion

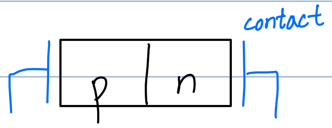


$$V_{out} = V_{in} + \overbrace{\alpha_1 V_{in}^2 + \alpha_2 V_{in}^3 + \cdots}^{\equiv D}$$

$$V_{out}' = D + A(V_{in} - V_{out}')$$

$$V_{out}' = \frac{\widetilde{D}}{1+A} + \cancel{\frac{A}{1+A}} V_{in} \approx V_{in} \quad \text{if } A >> 1$$

Diodes



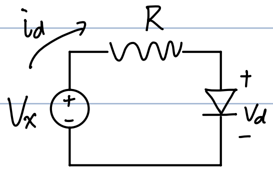
M-S potential fixed

p-n potential depends on V_{ext}

eq. $$i_d = \overset{\text{saturation, } \sim 10^{-14}}{I_s}\left(e^{V_d / \underset{1\le n\le 2}{n}\overset{\sim 26mV}{\phi_t}} \cancel{-1}\right)$$

$$\phi_t \equiv \frac{kT}{|q|}$$

E.

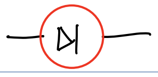


$$\frac{V_x - V_d}{R} = i_d = I_s e^{V_d / n\phi_t}$$

↳ transcendental :(

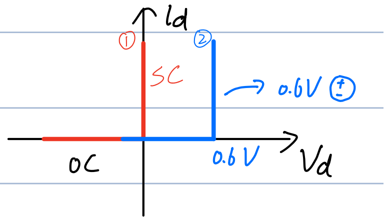
① $V_x \sim 100V \rightarrow V_d \approx 0$ in FB

→ ideal diode



② $V_x \sim 10V \rightarrow$ drop $\overset{Si}{0.6V}$, $\overset{Ge}{0.3}$, $\overset{LED}{1.5}$

→ fixed-drop model



③ $V_x \sim 1V \rightarrow$ full Shockley

a. numerical $$\frac{V_x - V_d}{R} = I_s e^{V_d / n\phi_t}$$ E. $R = 100$, $V_x = 1$

(1) guess $V_d = 0.5 V$

(2) $i_d \overset{Ohm}{=} \frac{1-0.5}{100} = 5mA$ → (4) $i_d = 3mA$ → . . .

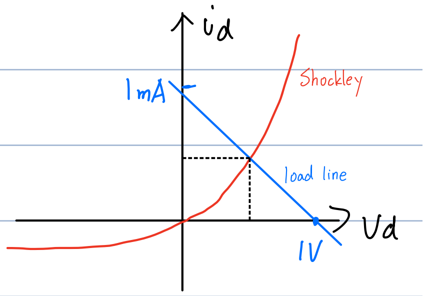
(3) $V_d \overset{Sch}{=} 0.7 V$ (5) $V_d = 0.687 V$

b. graphical Load line analysis

E. $R = 1k$, $V_x = 1$

load line: $\frac{1-V_d}{1k} = i_d$

find intersection w/ Shockley



Non-lin apps

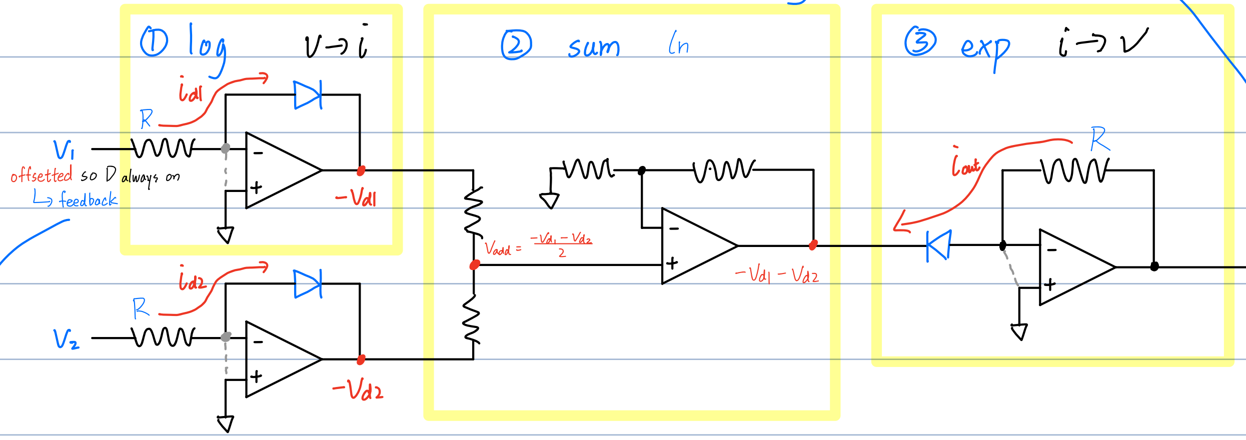
V_d	$\sim i_d$
0.5	2.25μ
1	505
10	10^{153}

1. log

① $V_{d1} = n\phi_t \ln\left(\frac{i_{d1}}{I_s} \overset{FB}{\cancel{+1}}\right)$

② $V_{d1} + V_{d2} = n\phi_t \ln \frac{i_{d1} i_{d2}}{I_s^2}$

③ $i_{out} = I_s e^{\frac{V_{d1}+V_{d2}}{n\phi_t}} = \frac{i_{d1}\, i_{d2}}{I_s}$ → analog ⊗

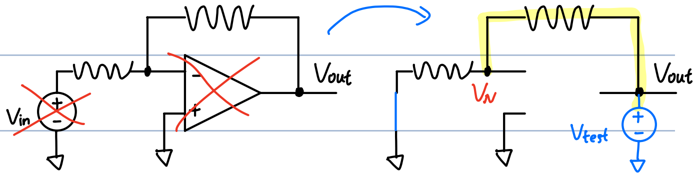


$V_{out} = \frac{V_{in1} V_{in2}}{R I_s}$

Input: offsetted sin waves. E. V_1: 9k, V_2: 10k $\overset{⊗}{\Rightarrow}$ V_{out}: 1k, 9k, 10k, 19k

Feedback check
1. Shut down source
2. Remove opamp, connect V_{out} to V_{test}
3. See if any of V_{test} appears at V_N

E.



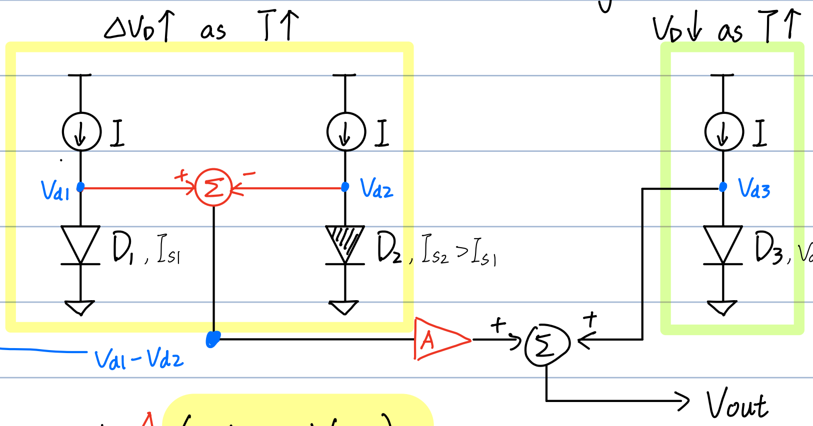
2. Temp

$V_d = n\phi_t \ln \frac{i_d}{I_s}$ $V_d = V_{d0} - 2\,{}^{mV}/_{K}\,(T - T_0)$

Bandgap ref. T↑ → V_D↓, but ΔV_D ↑, so we can weigh them and cancel

$V_d = n\phi_t \ln \frac{I}{I_s}$

$V_{d1} - V_{d2} = n\frac{kT}{q}\ln\frac{I_{s2}}{I_{s1}}$



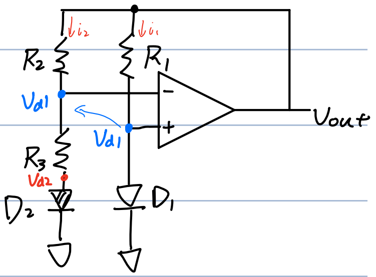
$V_{out} = V_{d3} + A(V_{d1} - V_{d2})$

$= V_{d0} - 2\,{}^{mV}/_{K}\,(T - T_0) + A\, n\frac{kT}{q}\ln\frac{I_{s2}}{I_{s1}}$

$= V_{d0} + 2\,{}^{mV}/_{K}\,T_0 + \underline{T\left(-2\,{}^{mV}/_{K} + \frac{Ank}{q}\ln\frac{I_{s2}}{I_{s1}}\right)}$

↳ Choose A so Vout indep. of T

Kujik bandgap ⊖ feed., since ⊖ fed, ⊕ clipped only by D_1.



$V_P = V_N \rightarrow i_1 = i_2$ if $R_1 = R_2$

$$V_{out} = i_2 R_2 \overset{V_X}{+ V_{d1}} \qquad i_2 = \frac{V_{d1} - V_{d2}}{R_3}$$

$$= \frac{R_2}{R_3}(V_{d1} - V_{d2}) + V_{d1}$$

$$= \frac{R_2}{R_3}\left(\frac{nkT}{|q|} \ln \frac{\cancel{i_1} I_{s2}}{\cancel{i_2} I_{s1}}\right) + V_0 - 2 mV/K\,(T - T_0)$$

$$= T\left[\frac{R_2}{R_3}\left(\frac{nk}{|q|} \ln \frac{I_{s2}}{I_{s1}}\right) - 2\,{}^{mV}/_K\right] + \left[\overset{\sim 1.2V}{V_0 + {}^{2mV}/_K\, T_0}\right]$$

Choose $\frac{R_2}{R_3}$ s.t. cancel

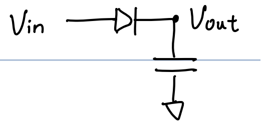
$n = 1.752$ for 1N4148

$$i_1 = \frac{\overset{1.2}{V_{out}} - \overset{0.7}{V_{D1}}}{R_1} \overset{want}{=} 5\ mA \xrightarrow{make} R_1 = R_2 = 250\,\Omega \rightarrow R_3 = 26.2\,\Omega$$

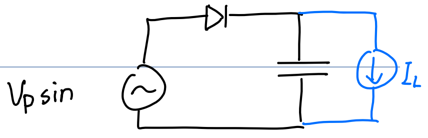
more accurately

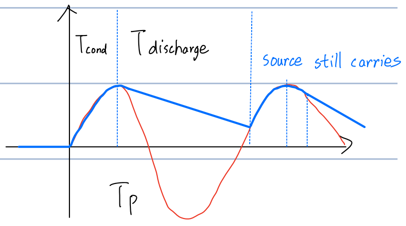
Find slope of V_{D1} vs. T → recalculate $R_3 = 30.1\,\Omega$

3. One-way → Peak detector



- Transformer $I_m = \frac{V_1}{j\omega L_1}$ → want $\omega \uparrow$ ($L \rightarrow \infty$ for ideal)
- Half wave charge → stay at peak, not rms





+ I_{Load} (E. battery charger)

$V \downarrow$ linear

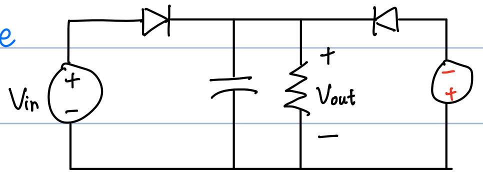
$$\Delta Q = \int_{T_{dis}} I_L\, dt$$

$$= T_{dis} I_L = C \Delta V \equiv V_R \text{ ripple}$$

$$C = \frac{T_{dis} I_L}{V_R}$$

if V_R small, $T_{dis} \approx T_p$, and for R load, discharge $\overset{exp}{\sim}$ lin, $I_L \approx \frac{V_p}{R_L}$

Full-wave



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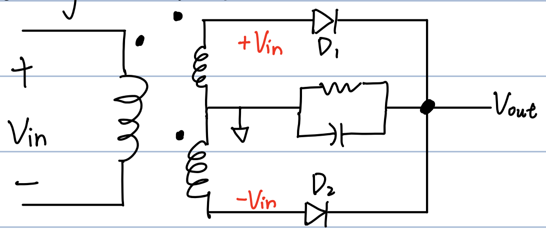
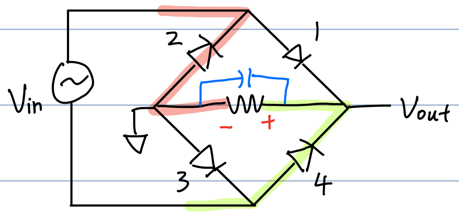


C only need hold half as long $C = \frac{I_L T_p/2}{V_r}$

peak inverse voltage

Can't breakdown $< 2 V_{in,p}$

Bridge



$V_{in} > 0 \rightarrow D_{1,3}$ on

$V_{in} < 0 \rightarrow D_{2,4}$ on

$< 1\ V_{in,p}$

I_D 1. avg. = avg I_{load} (half) / $\frac{1}{2}$ avg I_{load} (full)

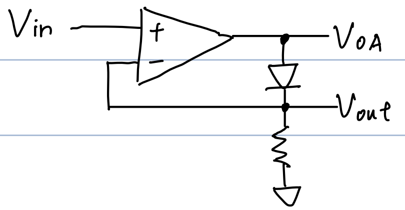
Long-term heating

2. peak $i_{peak} = C \overset{slope}{\frac{dV}{dt}} + I_{load}$

$\approx I_{load}\left(1 + 2\pi\sqrt{\frac{V_p}{2V_r}}\right)$

Precision rect.

aka super diode



$V_{in} > 0$, no drop, $V_{out} = V_{in} = V_{OA} - V_D$

$V_{in} < 0 \rightarrow$ pulled down to 0 $V_{OA} = V_{ss}$

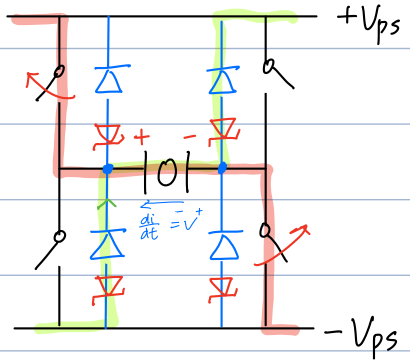
(inductive)

DC motor driver

↳ H-bridge

Zener

$\overset{\uparrow\uparrow}{V} = L\frac{di}{dt}$



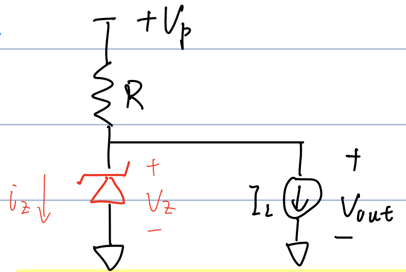
Switch off $\rightarrow V = L\frac{di}{dt}$

Need to tame: $V = -2V_{ps} - 2V_D - 2V_Z$

D required to block fwd current

Z for more drop.

Power supply

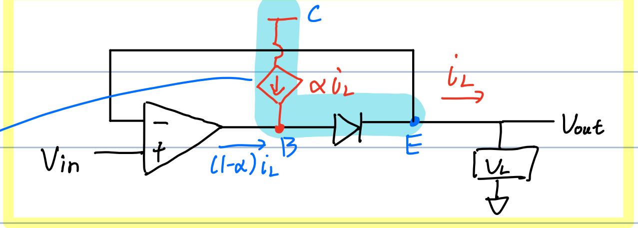


$V_{out} = V_Z$

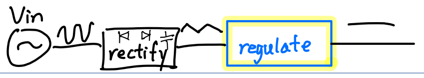
$I_L = \frac{V_p - V_Z}{R} - i_Z$, wasteful

Regulator

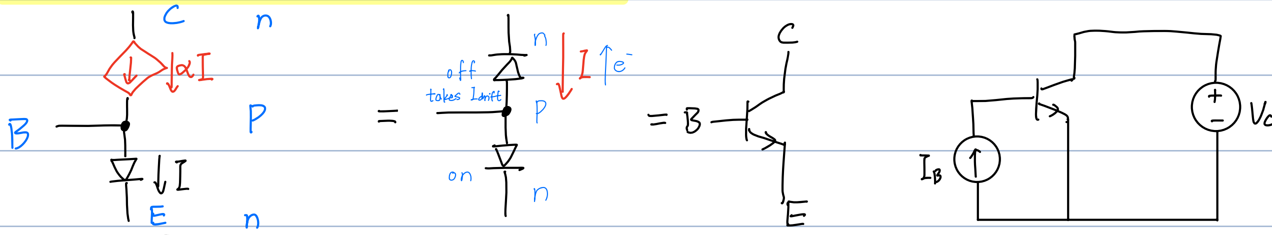
(from peak detector)



CCCS for more i_L drive

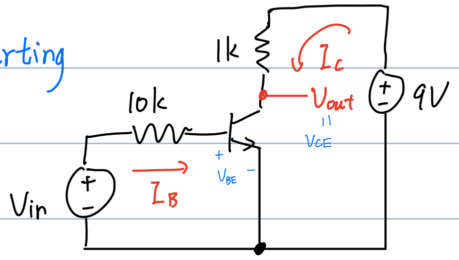


BJT



$\alpha \equiv \frac{I_C}{I_E}$ $\beta \equiv \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha}$

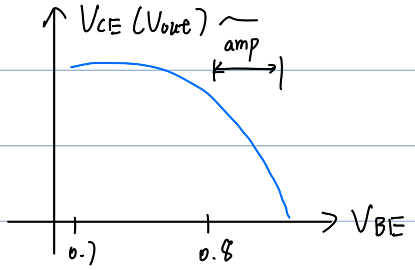
E. Inverting



load line $\left\{ I_c = \frac{9V - V_{CE}}{1k} \right.$, inverting $\Rightarrow$ Gsolve

trans. curve $I_c = f(V_{BE}, V_{CE})$

Want to stay in active (E. bias)



$$I_B = \frac{V_{in} - V_{BE}}{10k} = \frac{I_c}{\beta 100}$$

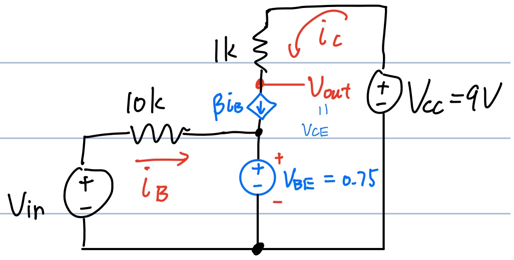
$$V_{in} = 100 I_c + V_{BE}$$

or plot Vin vs Vout → find slope

$I_B \xrightarrow{lin} I_c$ $V_{BE} \xrightarrow{exp} I_c$, better play w/ I_B

Approx. $V_{BE} = 0.75V$ (diode fixed drop)

↳ linear circuit



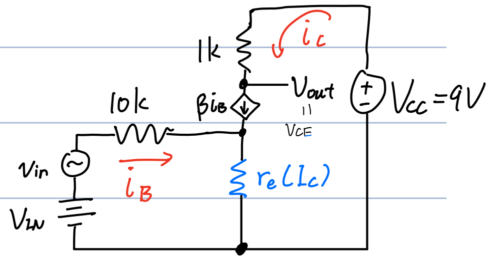
$$V_{out} = V_{cc} - i_c R_2 = V_{cc} - (\beta \frac{V_{in} - 0.75}{R_1}) R_2$$

$$= V_{cc} - \frac{R_2}{R_1} \beta (V_{in} - 0.75)$$

Now let $V_{in} = V_{DC} + \Delta V$

$$V_{out} = \underbrace{V_{cc} - \frac{R_2}{R_1} \beta (V_{DC} - 0.75)}_{V_{bias}} - \underbrace{\frac{R_1}{R_2} \beta \Delta V}_{V_{out}} = -10 \Delta V$$

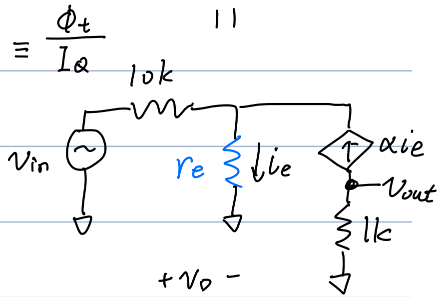
Small-sig $I_{\overset{quiescent}{Q}} = I_s e^{V_Q/\phi_t}$



$$V_D = V_Q + \overset{small}{\Delta V} \to i_D = I_Q e^{\Delta V/\phi_t}$$

$$= I_Q (1 + \overset{\Delta i}{\frac{\Delta V}{\phi_t}} + \cdots) \longrightarrow \text{lin!}$$

$$\Delta i = \frac{I_Q}{\phi_t} \Delta V \equiv \frac{\Delta V}{r_e} \longrightarrow \text{small-sig emitter } R \equiv \frac{\phi_t}{I_Q}$$



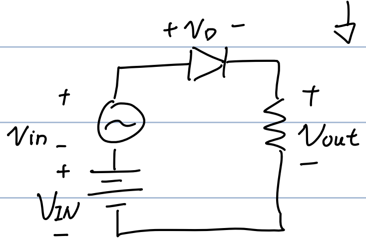
E. 1. Find bias point (nonlin) with V_{IN} (DC)

sim V_{OUT} , I_D

2. Turn off V_{IN} , replace D w/ r_d

$$v_{out} = v_{in} \left(\frac{R}{r_d + R} \right)$$

3. Add :) $V_{out} + v_{out}$



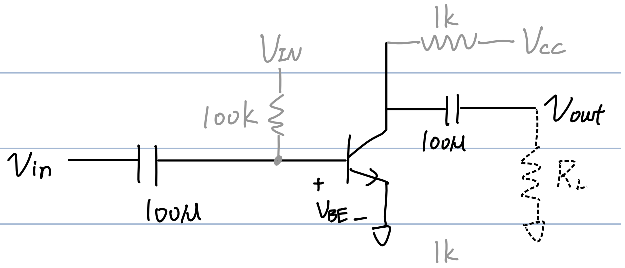
E. bias point $\xrightarrow{sim} I_E \longrightarrow r_e$

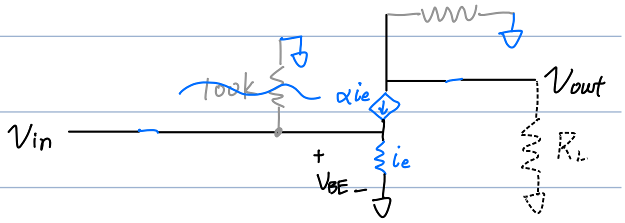
Bias point $V_{CE} = 9 - 1k\, I_C \overset{let}{=} 6\,V$

$$I_C = \beta \frac{V_{IN} \overset{diode}{-0.7}}{100k} \approx 3m$$

SS $v_{out} = (-\alpha i_e)\cdot 1k$

$= (-\alpha \frac{v_{in}}{r_e})\cdot 1k$





SS NPN

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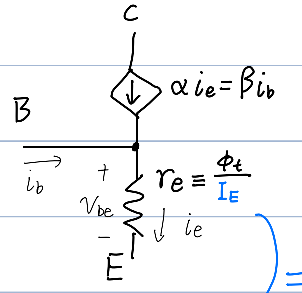
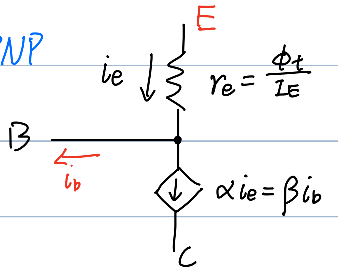


PNP



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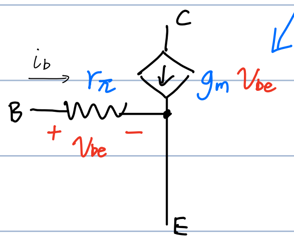
Hybrid π



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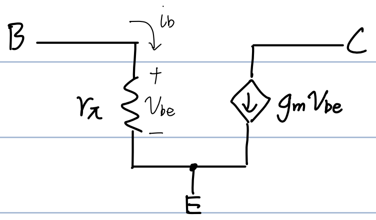


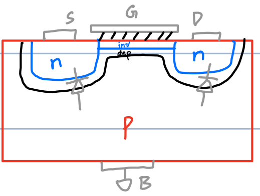
$\alpha i_e = i_c = g_m v_{be}$

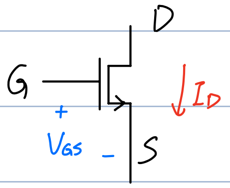
$g_m = \frac{\alpha}{r_e} = \frac{\alpha}{\phi_t/I_E} = \frac{I_c}{\phi_t}$

$i_e - i_c = i_b = \frac{v_{be}}{r_\pi}$

$r_\pi = r_e \frac{i_e}{i_b} = r_e(1+\beta) = \frac{\beta}{g_m}$

MOS Sym. device





$V_{GS} = 0$ Cutoff

$0 < V_{GS} < V_T$ Subthreshold

$V_{GS} > V_T$ Lin. small V_{DS} → R controlled by V_{GS}

$V_{DS}\uparrow$

$g \propto V_{GS} - V_t$

Triode channel pinched, R ↑, bad

☆ Sat. $V_{DS} \geq V_{GS} - V_T$, all pinched, but $\mathcal{E}\uparrow$, e^- drift thru

↳ VCCS

Sat. $i_D = \frac{1}{2} \overbrace{\mu C_{ox}}^{\equiv k'} \frac{W}{L} (V_{GS} - V_T)^2 \left(1 + \frac{V_{DS}}{V_A}\right)$ for nonflat sat.

Lin. $i_D = V_{DS} \underbrace{\left[k' \frac{W}{L} (V_{GS} - V_T)\right]}_{R}$

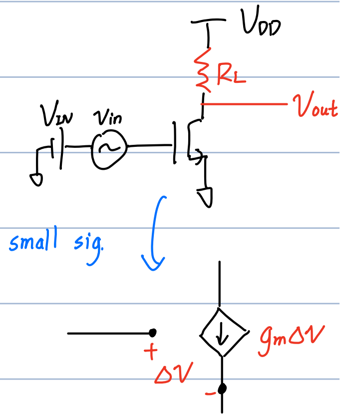
Amplifier Need V_{BIAS} for sat.

$i_D \overset{\cdots}{=} \underbrace{\frac{1}{2} k' \frac{W}{L} (\overset{\equiv V_{OD}}{V_{BIAS} - V_T})^2}_{\equiv I_{BIAS} \text{ region setup}} + \underbrace{k' \frac{W}{L} (V_{BIAS} - V_T)}_{\equiv g_m \text{, for amp.}} \Delta V + \cancel{\cdots \Delta V^2}$

↳ $= \sqrt{2 k' \frac{W}{L} I_{BIAS}}$

$= I_{BIAS} + g_m \Delta V$

$= \frac{1}{2} k' \frac{W}{L} V_{OD}^2 + g_m \Delta V$



$V_{OUT} = V_{DD} - \underbrace{\frac{1}{2} k' \frac{W}{L} V_{OD}^2 R_L}_{///}$ $V_{out} = -g_m v_{in} R_L$

$\equiv V_{DD} - V_{DROP}$ $\equiv -A_v v_{in}$

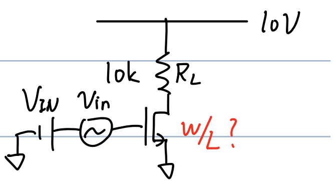
Design eq. $V_{OD} \overset{\cdots}{=} \frac{2V_{DROP}}{|A_v|}$ $\left(A_v = g_m R_L = k' \frac{W}{L} V_{OD}\right)$

E. $V_T = 1V$, $k' \frac{W}{L} = 44.4 \, mA/V^2$, $A_v = -20$, $\overset{V_{out}}{\text{swing}} = \pm 2V$, $V_{DD} = 10V$

Let $V_{OUT} = 5V \to V_{DROP} = 5V \to V_{OD} = \frac{2V_{DROP}}{|A_v|} = 0.5V \xrightarrow{+V_T} V_{GS} = 1.5V$

$|A_v| \overset{\cdots}{=} g_m R_L \to R_L = 900\Omega$

E. Find $\frac{W}{L}$, given $A_v = 20$, swing $\pm 2V$, $k' = 30 \, mA/V^2$, $V_T = 0.3V$



Choose $R_L = 10k$, $V_{DROP} = 5V \to I_D = 500 \mu A$

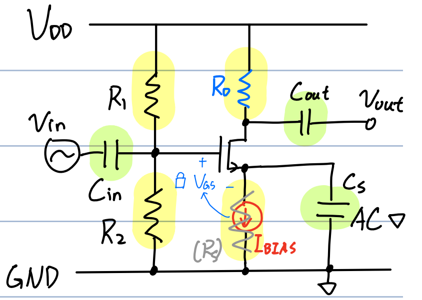
$|A_v| = g_m R_L = \sqrt{2k' \frac{W}{L} I_D} \; R_L$

$\frac{W}{L} \overset{\cdots}{=} 133.3$

Safer to bias I_D, not V_{GS} (V_{OD}), due to $\sqrt{\ }$

Bias ☆ I_{BIAS} pulls down V_S until V_{GS} is appropriate

indep. of V_T



E. $I_{BIAS} = R_S$ (variable) Given $V_T = 1V$, $k'\frac{W}{L} = 44.4\ ^{mA}/_{V^2}$, $V_{DD} = 10V$

want $A_v = -20$, swing $= \pm 2V$, $\underset{(3\sim7V)}{V_{DROP}} = 5V$, $f_{min} = 20\ Hz$

$\hookrightarrow$ $V_{OD} = \frac{2V_{DROP}}{|A_v|} = 0.5\ V$, $V_{GS} = V_{OD} + V_T = 1.5\ V$, $R_D = \frac{|A_v|}{g_m {\scriptstyle = k'\frac{W}{L}V_{OD}}} \overset{\cdots}{=} 900\Omega$, $I_D = \frac{1}{2}k'\frac{W}{L}V_{OD}^2 = 5.55mA$

Swing $V_{D,min} = 3V$, need $V_{GD} \lesssim V_T$ for sat.

$$V_{GS} = V_{GD} + V_{DS}$$

$$V_{OD} + \overset{SS\text{ small-sig}}{\cancel{V_T}} \geq \cancel{V_T} + V_{DS} \longrightarrow, V_{DS\ min} = V_{OD} = 0.5\ V$$

Bias Choose $V_S = 1V \longrightarrow R_S = \frac{V_S}{I_D} = 180\Omega$

$\overset{V_{GS}}{\curvearrowright}$ $V_G = 2.5V \longrightarrow$ make $R_1 = 75k$ $R_2 = 25\ k$ (10k ~ 100k)

Filter HPF to block DC

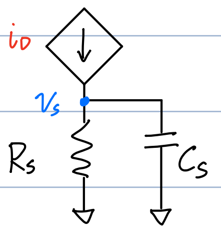
Cin $f_{in} = \frac{1}{2\pi C_{in}(R_1 || R_2)} = \frac{f_{min}}{10} \longrightarrow C_1 \overset{\cdots}{=} 4.24\ \mu F$

Cs $g_m(v_g - v_s) = i_D = v_s \frac{1+sR_SC_S}{R_S}$ (classical approach)

$$v_s \overset{\cdots}{=} v_g \frac{g_mR_S}{1+g_mR_S+sR_SC_S}$$

corner at $1+g_mR_S = \omega R_SC_S$

$C_S \overset{\cdots}{=} 2.2\ mF$ (R_S only 180Ω)



C_S will be much smaller if $R_S \uparrow$ ($\overset{\infty}{\hookrightarrow}$ ⓘ)

if no C_S, feedback at both AC / DC ($v_{gs}\uparrow \to i_d\uparrow \to v_s\uparrow \to v_{gs}\downarrow \to \cdots$)

$V_{out} = -(g_m v_{gs})R_D$ $\qquad v_{gs} \overset{?}{=} v_{in} - \underset{\text{not present before}}{g_m v_{gs} R_S} = \frac{v_{in}}{1+g_mR_S}$

$A_v = \frac{V_{out}}{V_{in}} \overset{\cdots}{=} \frac{-g_mR_D}{1+g_mR_S} \longrightarrow$ neg. feedback, but improves distortion at same output swing

$\approx -\frac{R_D}{R_S}$ if $g_mR_S >> 1$

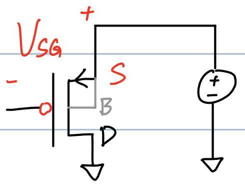
Cout depends on Rin of next stage

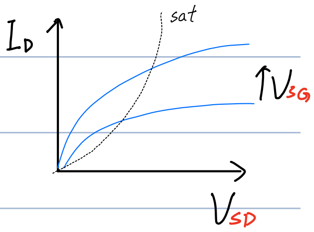
.model simpleFET NMOS (kp, W, L, Vto)

PMOS pull G down for I_D

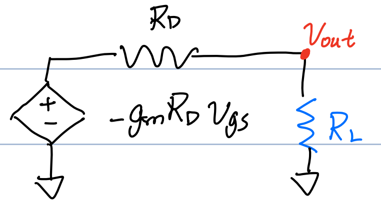
$I_D > 0$ for $V_{SG} > |V_{Tp}|$ and $V_{SD} > 0$

$$I_D = \frac{1}{2}k_p'\frac{W}{L}(V_{SG} - |V_{Tp}|)^2$$





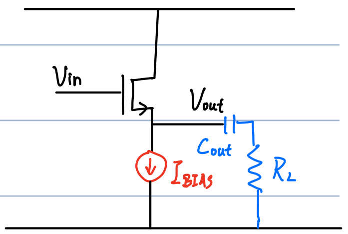
Load



$$|A_v| = g_m R_D \frac{R_L}{R_L + R_D}$$

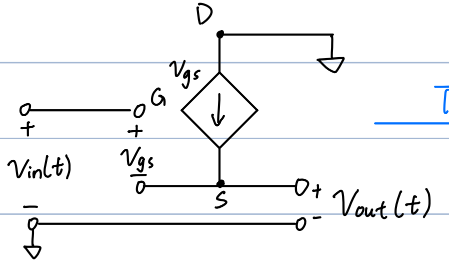
$$= g_m (R_D \parallel R_L) \qquad \text{(divided)}$$

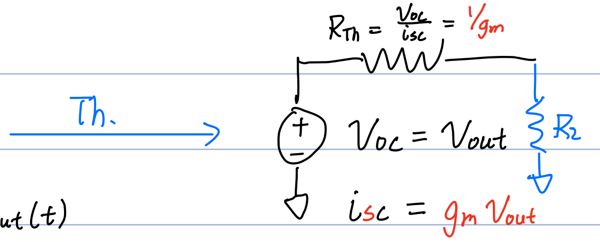
↳ Buffer (follower) (HW)



$I_D = \text{const.} \overset{ss}{\rightarrow} i_D = 0 \rightarrow V_{gs} = 0 \rightarrow V_{out} = V_{in}$

ss.





$R_{Th} = \frac{V_{oc}}{i_{sc}} = 1/g_m$

$V_{oc} = V_{out}$

$i_{sc} = g_m V_{out}$

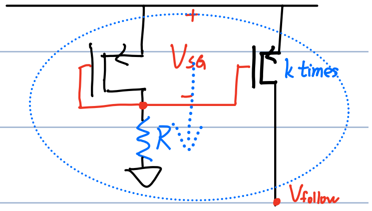
$$V_L = V_{in} \frac{R_L}{R_L + 1/g_m} \qquad 1/g_m \sim 10\,\Omega, \text{ small}$$

$$W_{corner} = \frac{1}{C_{out}(R_L + 1/g_m)}$$

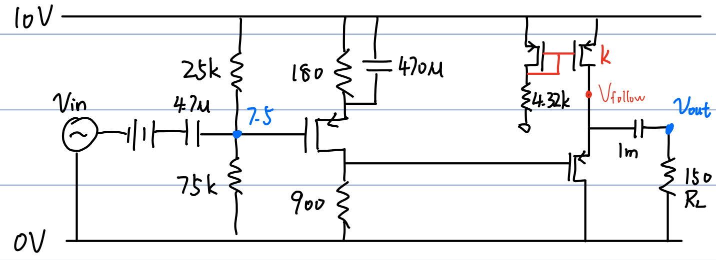
Make sure I_D swing remains active $\left(I_{BIAS} \pm \frac{V_{out}}{R_L}\right)$, else distorted

I_{BIAS} setup $(V_{SG} = V_{SD} \rightarrow I_D)$

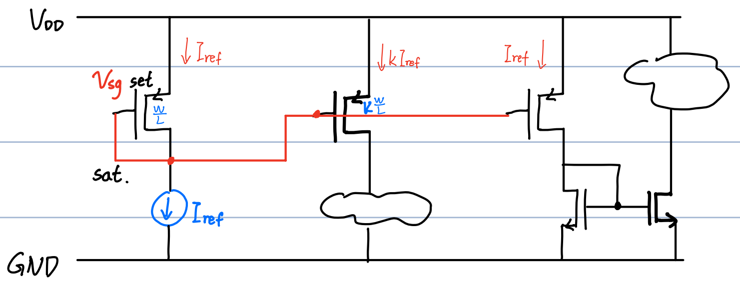
R just to set V_{SG} for k x PMOS → mirror



PMOS implementation



Current mirror I_{ref} to all analog cells

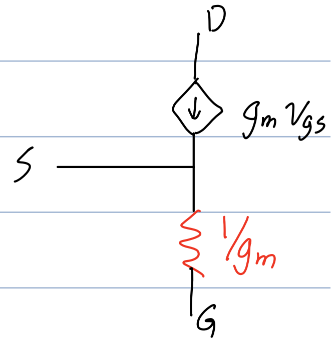


$I_{ref} \xrightarrow{\text{pulls}} V_{sg} = V_{sg,\,others} \rightarrow I_{others}$

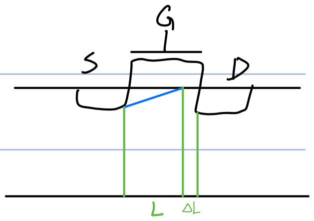
Need to ensure sat.

Alt ss. model (easier math)

$1/g_m$ cancels ◇, so i_g remains 0.



Channel width mod : non-ideal at sat.



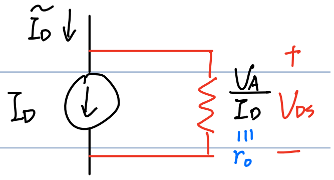
pinchoff: $\frac{W}{L} \rightarrow \frac{W}{L-\Delta L} = \frac{W}{L}\left(\frac{1}{1+\frac{\Delta L}{L}}\right) \approx \frac{W}{L}\left(1+\frac{\Delta L}{L}\right) \xrightarrow{\alpha} V_{DS}$

$$\tilde{I}_D = \frac{1}{2}k'\frac{W}{L}(V_{GS}-V_T)^2\left(1+\frac{\Delta L}{L}\right)$$

$$\equiv \frac{1}{2}k'\frac{W}{L}(V_{GS}-V_T)^2(1+\lambda V_{DS})$$

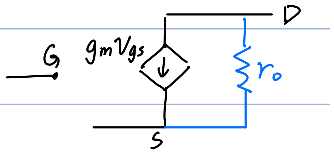
$\frac{1}{\lambda} \equiv V_A^{\sim 50V}$ (Early voltage)

$$\tilde{I}_D = I_D + \frac{I_D}{V_A}V_{DS} \longrightarrow \text{lin!}$$

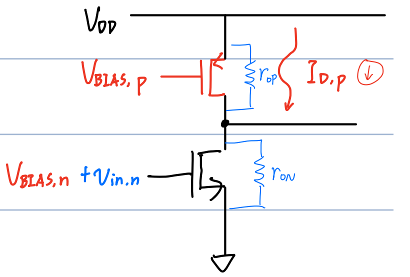


non-ideal I source (large sig)

ss $\left.\frac{\partial i_d}{\partial v_{ds}}\right|_{bias} = \frac{I_D}{V_A} \equiv \frac{1}{r_o}$ $r_o \sim 100k$ (want ∞)



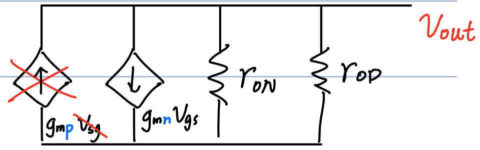
App. Current source load (more gain w/ r_o)



$v_{in,n} \rightarrow$ contradicting $\Delta i \rightarrow$ flows thru r_o (huge) $\rightarrow$ huge ΔV

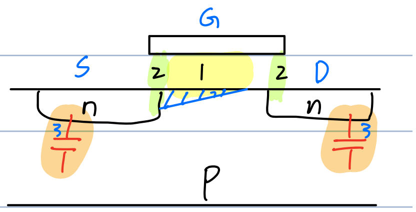
$$A_v = -g_{mn}(r_{on} \parallel r_{op})$$

$$|A_v| = k_n'\frac{W}{L}V_{OD_n}\frac{V_A}{2I_D} = \frac{V_A}{V_{OD}}$$



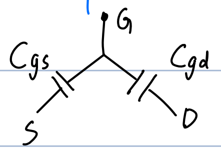
Parasitic Cap

1. gate 2. overlap ~~3. pn jct. cap~~



− Triode $C_{gs} = C_{sd} = \frac{1}{2} wL \overset{\text{cap/unit area}}{C_{ox}}$

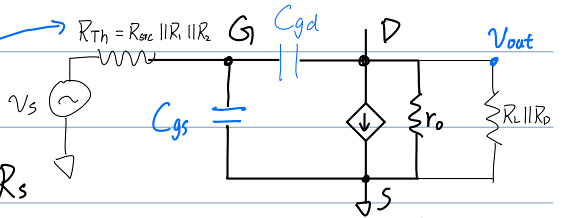
(tie S, D) E. op amp compensation

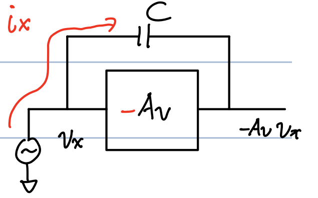


− Sat. pinched off at D side

$\overset{\sim pF}{C_{gs}} \doteq \frac{2}{3} wL\, C_{ox}$ $\overset{\sim fF}{C_{gd}}$ small, pinched

High freq. Turn bias $\xrightarrow{Th}$ $V_s = \frac{R_1 \parallel R_2}{R_{src} + R_1 \parallel R_2} V_{in}$ $R_{Th} = R_1 \parallel R_2 \parallel R_s$





Miller effect

$i_x = j\omega C(1+A_v)V_x$ (cap appears $\overset{\text{to gnd}}{}$ $1+A_v$ larger)

A_v can be made w/ opamp + pot $\overset{=}{\Rightarrow}$ var Cap

$C_{gd}' \rightarrow C_{gd}(1+g_m \cancel{R_o} \parallel R_L \parallel R_D)$ to gnd

$$f_{corner} = \frac{1}{2\pi RC} = \frac{1}{2\pi R_{Th}(C_{gs} + C_{gd}(1+g_m(R_L \parallel R_D)))} \quad (LPF)$$

2p|z $\rightarrow$ 1p